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FURTHER STUDIES ON CRITERIA FOR THE ONSET OF DYNAMICAL INSTABILITY IN GENERAL THREE-BODY SYSTEMS

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ABSTRACT

Results are presented from a variety of numerical experiments designed to further elucidate conditions under which self-gravitating three-body systems become dynamically unstable. We examined the stability of four types of orbital configurations: (1) circular, prograde, and coplanar orbits, (2) circular, retrograde, and coplanar orbits, (3) circular, direct, and inclined orbits, and (4) eccentric, direct, and coplanar orbits. Our experiments with circular, prograde, and coplanar orbits corroborate the stability criterion proposed by Graziani and Black (1981) as well as the generalized form of that criterion proposed by Black (1982). Our experiments with retrograde orbits were limited to systems where the tertiary body was significantly less massive than the binary bodies and in orbit about the binary as a whole (the so-called "outer planet" configuration). These retrograde systems are less stable than their prograde counterparts. Our results indicate that orbital inclination does not significantly affect stability for "outer planet" configurations, but that the stability of "inner planet" configurations, where the tertiary is in close orbit about one member of the binary, is noticeably less so once the relative orbital inclination $> 50°$. We find that the onset of dynamical instability is only weakly dependent on the eccentricity of either the binary or tertiary orbit as long as the mass of tertiary is comparable to the reduced mass of the binary. However, the dynamical stability of systems in which the tertiary mass is either much greater or much less than the reduced mass of the binary is a relatively strong function of orbital eccentricity.

I. INTRODUCTION

The question of whether a given three-body system (hereafter referred to as TBS) is dynamically stable is important for a wide variety of astronomical problems. Although some progress on this question has been made through the analytic studies of Szebehely and his coworkers (e.g., Szebehely 1976; Szebehely and McKenzie 1977; Szebehely and Zare 1977; Szebehely 1980), those studies did not provide a simple analytic criterion which could be used to determine whether a given system would be dynamically stable. Recently, Graziani and Black (1981) advanced such a criterion based on results from a series of numerical experiments. The Graziani-Black (hereafter referred to as GB) criterion was that a TBS was dynamically unstable if

$$\mu > \mu_{\text{crit}} = 0.175 \frac{\Delta^3}{(2-\Delta)^{3/2}}, \tag{1}$$

where μ and Δ are dimensionless mass and geometry parameters of a TBS given by:

$$\mu = \frac{(m_2+m_3)}{2m_1} \tag{2a}$$

and

$$\Delta = \frac{2(R_3-R_2)}{R_3+R_2}. \tag{2b}$$

The systems studied by GB were ones in which $m_2 = m_3 \leqslant m_1$. The geometry parameters R_2 and R_3 are defined in Fig. 1. Also shown in Fig. 1 are two other geometry parameters that are frequently used to characterize TBS: the semimajor axis of the binary orbit (a_1) and the peristron distance (q_2) of the tertiary body with respect to the barycenter of the binary pair. The operational definition of instability used by GB is that due to Laplace, viz. that one of the orbits develops clear (i.e., $\gtrsim 10$ percent) secular trends or becomes erratic. The term "instability" as used throughout the remainder of this paper has this definition unless it is specifically stated otherwise.

The GB study was confined to values of $\mu \leqslant 1$. Black (1982) has shown that the GB criterion is in excellent

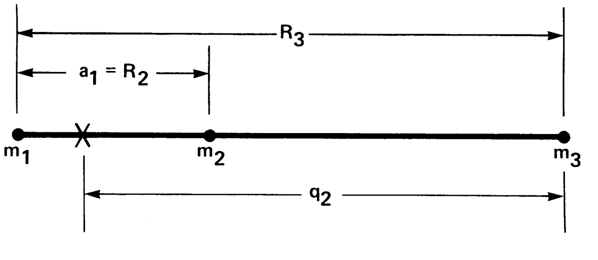

FIG. 1. Schematic representation of a coplanar three-body system as viewed perpendicular to the orbital plane and when the bodies are in a colinear configuration. This configuration serves to define the orbit parameters typically used to determine whether such a system is dynamically stable (see text for discussion). The center of mass of the $m_1 - m_2$ pair is denoted by the X.

agreement with results from analytical studies of the onset of instability in both the restricted and general three-body problem for circular, prograde, coplanar orbits. This agreement supports the definition of μ chosen by GB (μ could also be defined in terms of a geometric mean of m_2 and m_3, but such a definition gives $\mu \equiv 0$ for the restricted problem). We return to this point in Sec. III. Black also proposed the following extension of the GB criterion for values of $\mu \geqslant 1$: a TBS is unstable if

$$\mu > \mu_c = 0.083 \frac{\Delta^3}{(2-\Delta)^3} \qquad (3)$$

(m_1 and m_2 are always taken to be the binary pair in the TBS with m_1 being the more massive member of the binary). Equations (1) and (3) were derived from studies of co-revolving, coplanar, circular orbits. But whereas Eq. (1) was based on results from numerical experiments, Eq. (3) was based primarily on the general formalism of Szebehely and Zare (hereafter referred to as SZ). The primary purpose of this communication is to report results from a series of numerical experiments designed to examine the onset of dynamical instability in prograde, coplanar, circular TBS as well as in systems with either (a) retrograde, (b) inclined, or (c) eccentric orbits.

II. RESULTS

The results described here are from a series of numerical experiments on the evolution of TBS. The numerical code employed in this work is a modified version of the code due to Wielen (1964) (see GB for discussion of the code). Input parameters are the mass and the initial position and velocity of each body. Two integrals of the motion, total energy and angular momentum, were conserved to at least one part of 10^{10} during an experiment. All experiments were initiated with the three bodies in a colinear configuration (see Fig. 1).

Our experimental procedure was to fix μ and vary Δ, thereby locating a run of critical values of Δ as a function of μ. Experiments were run 100–1000 orbits of the longest period body, or until the existence of orbital instability was evident. Results are discussed for four general classes of experiments: (a) those with coplanar, direct, circular orbits, (b) those with coplanar, retrograde, circular orbits, (c) those with inclined, direct, circular and (d) those with coplanar, direct, elliptic orbits.

a) Systems with Coplanar Direct Circular Orbits

Shown in Fig. 2 are the $\mu-\Delta$ loci of several selected experiments. The $\mu-\Delta$ values pertain to the initial configuration of a system. Closed (open) symbols indicate systems which were found to be stable (unstable), while systems that showed marginal signs of instability are indicated by stippled symbols. Only those with the highest (lowest) Δ value which were unstable (stable) at the indicated μ value are plotted. We studied three generic types of TBS. In the nomenclature of Szebehely these

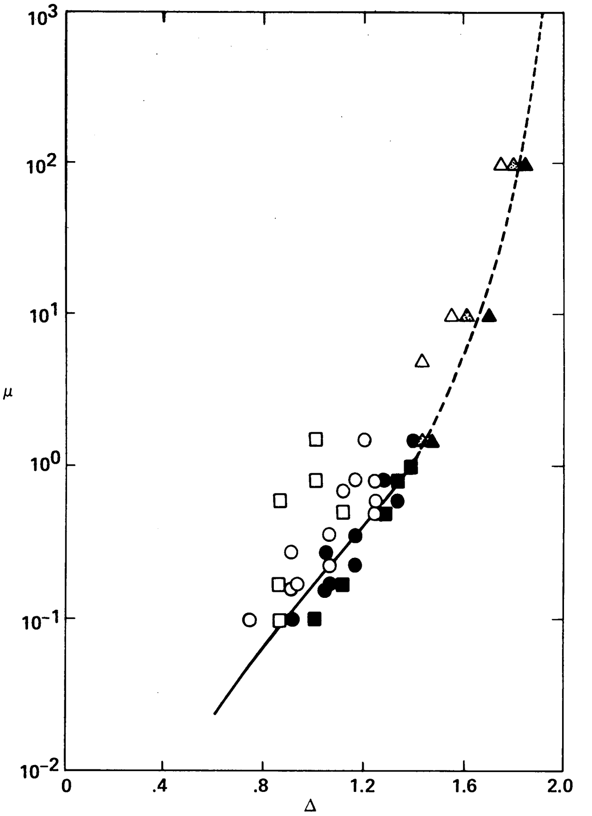

FIG. 2. A plot of μ–Δ (see text for definition of the dimensionless parameters) as defined by (1) the Graziani-Black (1981) three-body stability criterion (solid line), and the Black (1982) general three-body stability criterion (dashed line). Results from numerical experiments presented here as ●, ■, and ▲ represent, respectively, outer, inner, and satellite configurations. Open, stipple, and closed symbols indicate, respectively, unstable, marginally unstable, and stable systems.

are: outer planet, inner planet, and satellite. Outer planet refers to systems where $m_3 \leqslant \min(m_1, m_2)$, inner planet refers to systems where $m_2 \leqslant m_3 \leqslant m_1$, and satellite refers to systems where $m_3 \geqslant \max(m_1, m_2)$. Thus, satellite systems have $\mu \geqslant 1$, whereas inner and outer planet systems have $\mu \leqslant 1$. The symbols ●, ■, and ▲, respectively, are used to designate whether a system was outer planet, inner planet, or satellite.

The behavior of systems with $\mu \leqslant 1$ is generally in excellent agreement with that expected on the basis of the GB criterion. Our results support the findings of Black (1982) that a single stability criterion may be used for both the inner and outer planetary systems. Perhaps the best evidence for this is the data shown for $\mu = 0.5$. Both types of systems were stable at $\Delta = 1.27$ and unstable at $\Delta = 1.24$. Agreement between our results and those of GB is expected since both studies used modified versions of the same code. However, this study provides a more comprehensive data base against which the GB criterion can be tested because many more values of Δ in in the range of $1.0 \leqslant \Delta \leqslant 1.4$ were considered than was the case in the GB study.

Our results for $\mu > 1$ provide a test of Black's proposed extension of the GB stability criterion. The high-μ systems studied here are in very good agreement with that extension (the dashed line in Fig. 2). Our resolution in Δ is reasonably good for these experiments, particularly for the higher values of μ studied here. We did examine a system ($\mu = 5.0$, $\Delta = 1.43$) which would be stable according to the criterion given by Harrington (1977) but unstable according to the extended GB criterion; the system was unstable.

TBS experiments with $\mu > 1$ show a qualitative similarity to those with $\mu < 1$. The transition from stability to instability for systems with $\mu \gg 1$ (and $\mu \ll 1$) is very pronounced often leading to ejection of one member of the TBS. Systems with $\mu \sim 1$ tend to manifest instability in a much less pronounced way (e.g., ejection rarely occurred) than do systems with $\mu \gg 1$ or $\mu \ll 1$.

b) Systems with Coplanar, Retrograde, Circular Orbits

We tested the effect of retrograde binary orbits on the stability of the outer planet configuration for a TBS with $m_1 = m_2 = 0.5M_\odot$ and $m_3 = 10^{-3}M_\odot$ at $\mu - \Delta$ values which produced stable results with corresponding prograde motion. Our results show that higher Δ values are required to establish orbital stability in the retrograde case.

For example, a prograde $\mu = 0.5$ system is stable at $\Delta = 1.27$ while the same system is unstable if the binary orbit is counter that of the tertiary. A physical explanation for this result is that the retrograde system has less angular momentum than the prograde system if all other parameters are the same. The distance between the bodies, hence Δ, must be increased in the retrograde configuration to achieve the angular momentum equivalent to the prograde system at smaller values of Δ. Due to the nature of this study, ours is a sufficient, but not necessary condition for instability. Our findings support those of SZ that the sufficient condition for stability occurs at larger Δ values in retrograde systems than it does in their prograde counterparts.

c) Systems with Inclined, Direct, Circular Orbits

In order to test the dependence of orbital stability on the relative inclination of the orbital planes, a study of the inner and outer planet cases was made. This aspect of our experiments was confined to equal mass binary systems ($m_1 = m_2 = 0.5M_\odot$) where the "planet" had a mass of $10^{-3}\,M_\odot$. Relative inclinations of 25°, 50°, 75°, and 90° were considered with initial $\mu - \Delta$ values that ensured stability in the coplanar, direct, circular configuration. Our results show that orbital inclination has a strong effect on the inner planet model, with the onset of instability occurring between 50° and 75°. In the outer planet configuration no instability was detected, even near 90°.

d) Systems with Coplanar, Direct, Elliptical Orbits

Our experiments mainly studied the effect of eccentric binary orbits on the orbital stability of a planetary body in the inner planet configuration. The initial parameters provided for elliptical movement of the binary and circular movement of the planet about the primary component. The binary pair was initially at apogee. We started at values of Δ which give stable results in the circular case and tested eccentricities of $e = 0.2, 0.4$, and 0.6. The value of μ was varied by increasing the stellar mass ratios while keeping the planetary mass fixed at $10^{-3}\,M_\odot$. Qualitative results demonstrated that, for any given stellar mass ratio (fixed μ), Δ must increase as e increases to maintain stability. Furthermore, for fixed values of e we find that the critical value of Δ decreases as μ decreases. Comparing the results of SZ with ours, we found a more pronounced effect of eccentricity on the critical value of Δ for a given value of μ. For example, their study of the triple star case ($\mu = 1$) results in a change of 0.12 in the critical Δ value between the $e = 0$ and the $e = 0.5$ cases. We found an increase in the critical Δ value of at least 0.39 for $\mu = 0.1$ between the $e = 0$ and the $e = 0.6$ cases. We return to this point in Sec. III, emphasizing in a more general way the effect of orbital eccentricity on stability criteria.

III. DISCUSSION

The results presented here for circular, direct, coplanar orbits verify the criterion for dynamic instability in TBS with $\mu \leqslant 1$ as given by Graziani and Black (1981). They also verify the proposed extension of that criterion (Black 1982) to systems with $\mu \geqslant 1$. Our experiments with retrograde orbits were confined to cases where the total mass of the binary system, $m_1 + m_2$, was $1M_\odot$ and the mass ratio m_1/m_2 was 1.0, 3.0, and 5.0. In all cases, the mass of the tertiary body, m_3, was $10^{-3}M_\odot$. The orbits of these "outer-planet" cases were initially circular and coplanar. We find that retrograde systems of this type are *less* stable than their prograde counterparts, i.e., stability requires a higher Δ value for retrograde systems than for prograde systems with identical μ values. Hunter (1967) and Henon (1969) found retrograde systems to be *more* stable than their prograde counterparts for cases where $m_3 \geqslant m_1 + m_2$, i.e., the so-called "satellite" case. Because they considered a different type of TBS there is no disagreement between their results and ours. However, Harrington (1972, 1977) did investigate the effects of retrograde motion for the outer planet case and concluded that retrograde systems were *more* stable than their prograde counterparts. A close inspection of Harrington's results, particularly his 1977 paper where he did virtually the same experiment as we did with $m_1 = m_2$ and $m_3 = 10^{-3}M_\odot$ is instructive. Table I in that paper for the $e = 0$ outer-planet cases for corevolving ($I = 0$) and counterrevolving ($I = \pi$) systems shows that the lower limit for q_2/a_1 is *higher* for the $I = \pi$ case than for the $I = 0$ case. That is, the retro-

TABLE I. Values of $X(e_1, e_2)/X_0$ for various values of the eccentricities and for three different values of f_0: $f_0 = 10^2$ [Table 2(a)], $f_0 = 2$ [Table 1(b)], and $f_0 = 10^{-2}$ [Table 1(c)].

Table 1(a)

$e_2 \backslash e_1$	0	0.2	0.4	0.6	0.8
0	1	1	1	1	1
0.2	1.09	1.09	1.09	1.09	1.09
0.4	1.58	1.58	1.58	1.58	1.58
0.6	2.34	2.34	2.34	2.34	2.34
0.8	3.26	3.26	3.26	3.26	3.26

Table 1(b)

$e_2 \backslash e_1$	0	0.1	0.2	0.5	0.8
0	1	1.01	1.03	1.19	2.31
0.1	0.92	0.93	0.95	1.09	1.41
0.2	0.87	0.88	0.89	1.02	1.31
0.5	0.83	0.83	0.85	0.94	1.15
0.8	0.88	0.88	0.89	1.00	1.13

Table 1(c)

$e_2 \backslash e_1$	0	0.2	0.4	0.6	0.8
0	1	4.2	28	128	471
0.2	0.83	3.5	24	106	391
0.4	0.71	3.0	20	91	334
0.6	0.63	2.6	18	80	294
0.8	0.56	2.3	16	71	262

grade system is *less* stable than its prograde counterpart, which is what we find. The agreement between Harrington's work and our own is in a relative sense only; we find that higher q_2/a_1 values are required for stability. Harrington's other cases do seem to show that, for systems with *eccentric* orbits, retrograde orbits are more stable. We did no experiments of that type and are therefore unable to affirm experimentally this apparent reversal in relative stability.

Our experiments on the effects of relative orbital inclination in TBS show that if the members of the binary system have comparable masses, then inclined orbits are not significantly less stable than are coplanar orbits. If, however, the members of the binary system are of unequal mass, the inclined orbits tend to be significantly less stable than their coplanar counterparts. In cases where one member of the binary has a mass comparable to that of Jupiter we found clear signs of instability occurring when the relative inclination of orbital planes exceeds $\sim 50°$. It is of interest to compare our results on the stability of TBS with inclined orbits with the findings of other investigators. Hunter (1967) found that the *more* inclined the orbits in models of the Sun-Jupiter satellite, the more stable they were. Harrington (1972) found that there was no dependence of instability on the relative inclination of orbits for a system where $m_1 = m_2 = m_3$. As we consider only the so-called "inner" and "outer" planet cases, our results cannot be directly compared with those of Hunter. Although Harrington used different relative masses than we did in our experiments, close examination of the details of the two experiments does not indicate an inconsistency in the results. Harrington considered relative inclinations of 0°, 45°, 90°, 135°, and 180°, or 0°, 45°, and 90° if one considers only prograde motion. In contrast, we considered relative inclinations of 0°, 25°, 50°, 75°, and 90°. Harrington found that systems with relative inclinations of 45° were no less stable than coplanar ($i = 0°$) systems. Our results for the outer-planet case extend this result to $i = 75°$. We do find evidence that inner-planet systems with $i \geqslant 50°$ are less stable than their coplanar counterparts. Because of differences in resolution in inclination angles between our experiments and those of Harrington, our results are not inconsistent with his.

As noted in the previous section, our experiments with TBS in which a relatively massive tertiary body is in orbit about an eccentric binary, show that the critical Δ value for such systems is sensitive to the eccentricity of the binary orbit. Given the fact that most stellar TBS have eccentric orbits, we consider here the effects of orbital eccentricity on the onset of instability in more detail.

Using the approach of SZ, the following equation can be used to determine the critical value of Δ for a prograde TBS:

$$[1 + f(m_i,e_2)x^{-1}][1 + g(m_i,e_1,e_2)x^{-1/2}]^2 = h(m_i,e_2,S_{cr})x^{-1}, \tag{4}$$

where e_1 and e_2 are, respectively, the eccentricity of the binary and tertiary orbits,

$$f(m_i,e_2) = \frac{m_3(m_1+m_2)(1-e_2)}{m_1 m_2}, \tag{4a}$$

$$g(m_i,e_1,e_2) = \frac{m_1 m_2}{m_3}\left[\frac{m_1+m_2+m_3}{(m_1+m_2)^3}\right]^{1/2} \times \left[\frac{1-e_1^2}{1+e_2}\right]^{1/2}, \tag{4b}$$

and

$$h(m_i,e_2,S_{cr}) = \left(\frac{2}{243}\right) S_{cr} \frac{(m_1+m_2+m_3)^6}{m_1 m_2 (m_1+m_2)^2 m_3^2}\left[\frac{1}{1+e_2}\right]. \tag{4c}$$

The parameter $X = q_2/a_1 = (R_3/R_2) - m_2/(m_1+m_2)$ and can thus be used to determine Δ. S_{cr} is determined solely by the masses in a TBS (see SZ for details). In general, one must solve a quintic equation to determine S_{cr}. It should be emphasized that although Eq. (4) is based on approximation of the general three-body interactions by two-body interactions and is thus not valid when $e_2 \simeq 1$ or when $e_1 \simeq 1$, it does provide insight into the effects of nonzero e_1 and e_2 on the onset of instability.

Equation (3) admits closed form solutions only when either e_1 or e_2 is unity.

$$x(1,e_2) = h(m_i,e_2,S_{cr}) - f(m_i,e_2) \tag{5}$$

and

$$X(e_1,1) = \{[h(m_i 1,S_{cr})]^{1/2} - g(m_i,e_1,1)\}^2. \tag{6}$$

The critical x value, x_0, for circular orbits satisfies

$$(1 + f_0 x_0^{-1})(1 + g_0 x_0^{-1/2})^2 = h_0 x_0^{-1}, \quad (7)$$

where we have set

$$f(m_i, 0) = f_0,$$

$$g(m_i, 0, 0) = g_0,$$

and

$$h(m_i, 0, S_{cr}) = h_0$$

for notational simplicity. Further,

$$f_0 g_0 = [1 + m_3/(m_1 + m_2)]^{1/2} \geqslant 1$$

which coupled with the definition of f_0 indicates that if $f_0 \gg 1$ then $g_0 \ll 1$ and conversely. We can thus characterize solutions to Eq. (4) in terms of whether $f_0 \gg 1$, ~ 1, or $\ll 1$. The physical quantity corresponding to f_0 can be found by recalling that $f_0 = m_3/\mu_B$, where $\mu_B = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the binary pair. Thus the conditions $f_0 \gg 1$, ~ 1, $\ll 1$ correspond, respectively, to the physical conditions $m_3 \gg \mu_B$, $m_3 \sim \mu_B$ and $m_3 \ll \mu_B$.

The behavior of systems for which $f_0 \gg 1$ is given by

$$\frac{x(e_1, e_2)}{x_0} \simeq \frac{(1 + e_2^2 f_0 x_0^{-1})}{(1 + e_2)}. \quad (8)$$

Equation (8) is valid as long as

$$x^{1/2}(e_1, e_2) \ll \frac{(1 - e_2)(1 + e_2)^{1/2} f_0^2}{2[1 + m_3/(m_1 + m_2)]^{1/2}(1 - e_1^2)^{1/2}}$$

(i.e., as long as

$$f_0(1 - e_2)/x(e_1, e_2) \gg 2 g_0 (1 - e_1^2)^{1/2} / [(1 + e_2) x(e_1, e_2)]^{1/2}).$$

Note that the ratio $x(e_1, e_2)/x_0$ is independent of e_1 and that it has a minimum value of $\simeq 1 - x_0/4f_0$ at $\tilde{e}_2 = [1 + x_0/f_0]^{1/2} - 1 \simeq x_0/2f_0$. Once $e_2 > \tilde{e}_2$ the ratio $x(e_1, e_2)/x_0$ increases rapidly with increasing e_2. The behavior of TBS in which $f_0 \ll 1$ is given by

$$\frac{x(e_1, e_2)}{x_0} \simeq \frac{\{1 + \alpha[1 - (1 - e_1^2)^{1/2}]\}^2}{1 + e_2}, \quad (9)$$

where

$$\alpha = \frac{g_0}{x_0^{1/2}} = \frac{1}{f_0}\left[\frac{1 + m_3/(m_1 + m_2)}{x_0}\right]^{1/2} > 1.$$

Both Eqs. (8) and (9) have a $(1 + e_2)^{-1}$ dependence. As expected, there is a strong dependence of $x(e_1, e_2)/x_0$ on e_1 [Eq. (9)] and on e_2 [Eq. (8)] in prograde TBS where $m_3 \gg \mu_B$. The behavior of $x_0(e_1, e_2)/x_0$ for $f_0 = 10^2$ and $f_0 = 10^{-2}$ is shown in Tables 1(a) and 1(c), respectively. SZ have obtained solutions to Eq. (4) for the case where $m_1 = m_2 = m_3$ (i.e., $m_3 \sim \mu_B$). Those solutions are given in Table II and Fig. 2 of their paper as well as in Table 1(b) of this paper.

It is apparent from Table I and the above discussion that the critical q_2/a_1 of Δ values for the onset of instability in prograde TBS can be significantly increased if either $m_3 \gg \mu_B$ or $m_3 \ll \mu_B$. However, for TBS in which $m_3 \sim \mu_B$ (more precisely for TBS in which $0.3 \lesssim f_0 \lesssim 10$) there is only relatively minor change ($\simeq$25 percent) in the critical value of Δ even for large values of e_1 and/or e_2. The eight triple star systems discussed by Black (1982) have f_0 values ranging from a high of 5.5 (ζ Aqr) to a low of 0.35 (λ Tau). Consequently, the stability of those systems should be reasonably well described by the GB criterion for circular orbits (coincidentally $e_1 = e_2 \simeq 0$ for λ Tau). This further supports Black's (1982) assertion that the observed absence of stellar triple systems with $q_2/a_1 \simeq 5$ is due to instability rather than cosmogonic (i.e., formation) processes.

It is important to recognize that the question of what conditions produce a dynamically unstable three-body system is dependent to some extent on the nature of the system. For example, Hunter (1967) states that retrograde Jupiter satellites are more stable than their prograde counterparts if the orbits are coplanar, but that prograde satellites become more stable than their retrograde counterparts for orbits inclined at angles $\gtrsim 45°$. Similar *qualitative* differences are found for the so-called inner and outer planet cases. However, on a *quantitative* level it appears that over a very wide range of relative masses and orbital parameters (e.g., inclination, eccentricity) one can use the simple criterion given by Graziani and Black (1981) and by Black (1982) to determine whether a given system would be dynamically unstable.

We wish to acknowledge the many contributions by Dr. Szebehely and his co-workers to the three-body problem. We are grateful for their insight into, and valuable constraints on, the essential nature of this problem. We also appreciate many helpful discussions with Drs. F. Graziani and T. Ackerman. This paper has benefited from a careful and thoughtful review by an anonymous referee whom we wish to thank.

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